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Stellar Seismology, Stellar Ages and the Cosmological Constant

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Abstract. Solar seismology has allowed precision measurements of both the static and dynamic structure of our local star, the Sun. In the near future, seismology of solar-like stars of different ages and masses, necessarily restricted by angular resolution to low l-modes, will allow studies of the internal structure of stars at various stages of evolution. Such studies will test not only the theory of stellar evolution, but also allow the determination of ages of stars from the helium content in their cores. Such observations can be made photometrically from space, but also spectroscopically from the ground. We outline ground-based schemes. By correlating the external properties of nearby stars with their internal properties, it will be possible to extend local studies to distant open and globular clusters, and thereby to obtain an age of the Universe, based on many stars. The combination of the age, the density parameter Ω and Hubble's constant will allow strong limits to be placed on the cosmological constant.

1. Introduction

Recent measurements of distant, type Ia supernovae, taken as standard candles (Perlmutter *et al.*, 1999; Riess *et al.*, 1998), have been interpreted as implying an accelerating expansion of the Universe and demanding the existence of a cosmological constant and, presumably, a new fundamental interaction (Zlatev, Wang & Steinhardt, 1999). The implication of such interpretations is profound, not just for cosmology, but for physics at large. Clearly, such extraordinary claims require extraordinarily good evidence, preferably by as many, independent means as possible. A possible method of verification is the determination of the ages of stars in our own galaxy, at $z = 0$, and, thereby, a lower limit to the age of the Universe. With a Hubble constant $H_o = 67$km s^{-1} Mpc^{-1} and a density parameter $\Omega = 1$, the age of the Universe is 9.6 Ga if the cosmological constant, $\Lambda = 0$. Adopting the currently fashionable cosmological vacuum energy contribution of $\Omega_\Lambda = 0.7$ demands an age of 13.8 Ga.

To measure stellar ages requires a calibration of the scale on which such age determinations are based – a calibration of the theory of stellar evolution, which has usually been taken as sacrosanct. The Sun provides a calibration, albeit at one point only in the parameter space of mass, composition and age.

An external calibration, using only the readily measured external parameters has, of course, been used in the past. An internal calibration, using parameters of the solar (stellar) core, is also, in principle, possible. This provides a near-direct measure of the molecular weight in the core and, thereby, measure of the helium content. If the original amount of helium is presumed to be known and if gravitational settling of helium is allowed for, the current helium content provides a measure of the amount of thermonuclear conversion of hydrogen into helium and therefore, with a known luminosity, the age of the star.

Stellar seismology, an extension of the successful solar seismology, is capable of providing this information both in principle and, in the near future, in practice for stars over a range of ages. There is prospect that a reasonable zero-age calibration point, in addition to that given by the Sun, could be provided by a very young star, preferably one with known mass and composition. The absolute error in the theoretically determined age is then likely to be sufficiently small. This, via a cosmological aeon ladder, ought to allow us to determine the ages of old stars and also of those in old open and globular clusters.

We propose to use the existing seismological calibration of the Sun to bootstrap and thereby determine the ages of stars, and thus check the theory of stellar evolution over a wide range of stellar masses, internal composition and importantly age. Such an undertaking, using stellar seismology, is feasible using current technology and is underpinned by extensive work that has gone into establishing the acoustic eigenmode spectrum of the Sun over the last two decades.

Here, we outline, albeit very sketchily, the relevant key elements of solar seismology, its extension to stars, the sensitivity to age of the relevant eigenfrequency separations and the feasibility of those measurements from a space platform as well as from the ground.

2. Solar Seismology

Electromagnetic spectroscopy of the eigenmodes of the atom has provided a wealth of tools with which to probe Nature at large over the last one and a half centuries. It would seem that the detailed study of the acoustic spectrum of a star, (see Figure 1), and in particular the study of eigenfrequencies, shows equal promise. Unlike many other parameters pertaining to stellar systems, eigenfrequencies are both precise *and* accurate. Frequency measurements with an accuracy of up to a few parts per million, relevant in our context, have been made with no systematic error. Some statistical errors are, however, present: in particular, those arising from the finite duration of any observation, the finite lifetime of the eigenmode itself, daily aliases and additional amplitude noise. Inspite of these, it was possible already back in 1981 (Claverie *et al.*, 1981) to determine the accoustic eigen-frequencies of the individual low $l, l = \mathit{0}, \mathit{1}, \mathit{2}, \mathit{3}$ modes of the global Sun (Sun as star) to better than 1 part in 4000. Currently, the accuracy attained is better than 1 in 10^5 (Chaplin *et al.*, 1998).

Individual eigenfrequencies provide integral measures of the velocity of sound, and thus of $\sqrt{\gamma kT/m}$ over the ray path of the sound wave, with different modes probing different depths. At present the accuracy to which the eigenfrequencies can be measured far outstrips the level to which theoretical models and observational data agree. Thus, little useful science can, at present, be extracted

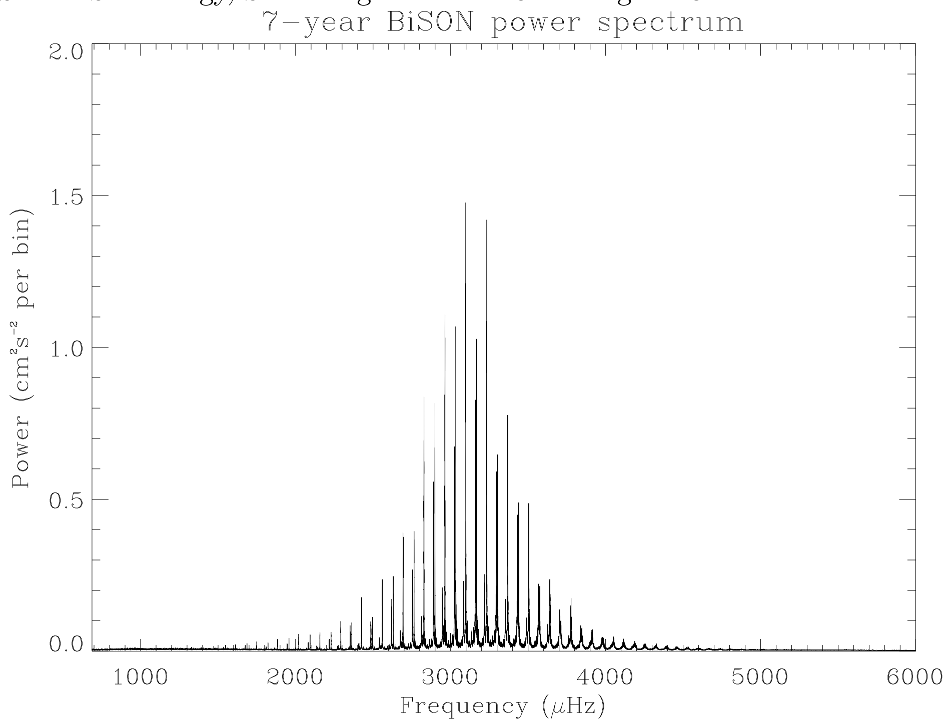

Figure 1. A power spectrum generated from 7 years of data accummulated from 6 stations of BiSON, the Birmingham Solar Oscillation Network (Chaplin *et al.*, 1996)

from direct comparisons between observed eigenfrequencies and their 'postdiction' by theoretical models. A large part of this problem is believed to be the incomplete treatment of the contributions of the surface layers to the travel time of sound waves. By taking *differences* between eigenfrequencies this problem can be substantially eliminated. Differential information gleaned by measuring frequency separations includes: (a) the separation between successive harmonics of the radial and low l non-radial modes - this large separation, Δ, provides a measure of the time taken by sound waves to traverse the diameter of the star – $\Delta \sim (2\int dr/c)^{-1}$. (b) the small separation, d_o between radial and quadrupole p modes – closely spaced in frequency; these modes share a very similar path in the outer regions of the Sun, but penetrate to different depths in the core, and thus measure core properties. (c) an even smaller splitting due to the removal of degeneracy of the non-radial eigenmodes by rotation, the rotational splitting: the number of components, $2l + 1$, can be used to identify the mode, with their spacing providing an integral measure of the internal rotation.

The use of such differencing methods is common in other branches of physics. Perhaps the most familiar example can be found in an atomic analogue: from fine-structure, hyperfine-structure and isotope shifts of observed optical atomic transitions it is possible to infer details about both atomic and nuclear properties that are very much more subtle than could be inferred from theoretical modelling of the atomic term values using Dirac's equation. In a similar way d_o provides a sensitive measure of the physical conditions - temperature and molecular weight – in the core of the star. As the conditions vary with mass and

age, so too does the separation – in the Sun, its value is close to 9μHz. As a star evolves, hydrogen is converted into helium by thermonuclear fusion and, in the absence of mixing, the helium fraction in the nuclear active core increases. The resultant increase in the molecular weight in those regions sampled more by the radial than the quadrupole modes, results in a decrease in the velocity of sound and thus in the value of d_o. The variation of both d_o and Δ with evolutionary age for stars of different masses has been evaluated by J.Christensen-Dalsgaard (1984,1988) – for a solar-type star at the midpoint of its life, as is the Sun, the sensitivity of the fine-spacing is near 1μHz/Ga. Thus, the accuracies to which Δ and d_o were measured in the Sun already back in 1981, about 0.3μHz, correspond to an error in the determination of the solar age of 0.3 Ga. Current measurements have errors some twenty times smaller. Thus, it is clear that it is model-dependent, and NOT observational, error that will dominate any error-budget.

3. Calibration and pitfalls

The calibration of any cosmological aeon glass is full of pitfalls. Whilst the value for the age of the meteorite-Earth system, determined using radioactive decays of isotopes of K, Rb, Sm, Lu, Re, Th and U, has remained within $\pm$0.01 Ga of the value of 4.55 Ga over the last 45 years (Dalrymple, 1994), the value for the age of the (oldest) globular clusters has varied over the range of 5 to 25 Ga over the same time interval (Sandage, 1962; Gamow, 1964). This in itself serves as an indication that absolute calibration of the theory of the structure and evolution of stars is essential. Such calibration of the basic physics has been provided by solar seismology for the last twenty years, while theoretical solar models have been adjusted repeatedly over that time to approximate to the measured eigenfrequencies. The calibration of a model, solar or otherwise, at time t, means that we adjust the proposed model either **quantitatively** by adjusting parameters or **qualitatively** by means of the addition of another physical mechanism such that model age and radioactive age agree exactly. Two examples, however, may serve to demonstrate some of the pitfalls in such work. The first comes from the discovery measurements of global solar oscillations (Claverie *et al.*, 1979). One of us, GRI (Isaak, 1980) used the extreme high frequency end of the eigenfrequencies of the solar models of Iben & Mahaffy (1976) and Christensen-Dalsgaard, Gough & Morgan (1979), models which had been constructed in response to the apparently successful results of a long-term campaign to search for the fundamental radial and low n non-radial eigenmodes in 1974-5. The theoretical models did not predict the excitation of high n, low l acoustic modes. It was fortuitous that the published values extended into the region in which, totally unexpectedly (Unno *et al.*, 1979), global solar oscillations were discovered. GRI deduced, by interpolation, that the best fit between Δ and the models implied a helium abundance of less than $Y = 0.17$, in agreement with the average solar wind value *but* the observational data, thought to be accurate to the 0.3% level, were indeed correct, however the model was wrong and required substantial quantitative adjustments. Thus, the potentially fundamental cosmological implication of this very low Y inference was false.

A second example is based on the much improved measurements of d_o which BiSON (Birmingham Solar Oscillations Network) has been producing consistently since the early 1990's. Once again, GRI compared the measured values of d_o with the much improved models of Christensen-Dalsgaard, concluding that the fit was poor – assigning an age of 5.2 Ga to the Sun would have provided an excellent match between model and data, however this would have been totally inconsistent with the radioactively-determined solar age, which by then was very secure. In principle, the Sun could be older than the rest of the Solar System, however, the presence of daughter products of short-lived isotopes such as ^{26}Al, ^{107}Pd and ^{41}Ca in meteorites suggests strongly that the meteorite formation was coeval with the Sun (Guenther, 1989, Bahcall *et al.*,1995), presumably triggered by a supernova explosion. Models suggest that it takes around 50 Ma for a molecular cloud to collapse onto the zero age main sequence (ZAMS) (Schwarzschild 1958, Iben 1965). Thus, the age of the Sun, measured from its arrival on the ZAMS is 4.56-0.05=4.51 Ga, with an uncertainty which would seem to be at most 0.02 Ga. GRI preferred the radioactive age to the seismological age and speculated, in common with others (Christensen-Dalsgaard, Proffit & Thompson, 1993), that helium settled gravitationally (Isaak, 1993), with the idea shortly afterwards that heavier elements (Proffitt, 1994) settled also. These *qualitatively new* additions to the physics of the seismological Sun can very nearly, but not perfectly, reproduce the measurements (Elsworth *et al.*, 1995, Chaplin *et al.*, 1997). A cynical reader might wonder as to the relevance of the above: clearly, the measurements are of an integral nature and no unique interpretation is possible. We suggest, however, that the study of solar seismology has substantially enriched the theory of stellar structure and evolution, beyond that first proposed by Eddington of 1928, by confronting theory with accurate observation. The future of stellar seismology has even greater potential, if appropriate financial resources were to be made available.

4. Can one detect stellar oscillations?

Soon after the discovery of global solar oscillations (Claverie *et al.*, 1979), it was pointed out by GRI (Isaak 1980) that the detection of stellar oscillations of the size seen on the Sun was readily achieved in one of two ways: (a) by measuring spectroscopic velocities using large flux collectors (b) by photometric measurements using a photometer and very modest optical telescopes situated above the transparency fluctuations and the turbulence of the terrestrial atmosphere.

Two years later, GRI also suggested a stellar seismology mission to the European Space Agency; however neither this mission, nor any of its many successors over the last 18 years, have been launched.

Can stellar oscillations really be detected? Here, we assess the feasibility of the detection of solar-like modes in main-sequence stars. We assume (a) oscillation amplitudes of a size comparable to those seen on the Sun – there is some evidence to suggest (Houdek *et al.*, 1995) that the more evolved a star, the larger the amplitude of oscillation. We adopt, however, a conservative amplitude of $\sim 10cm/s$ in velocity, and 2 parts per million in intensity (b) an extension of spectroscopic measurements from the Sun to the brightest local stars is difficult as the fluxes of the Sun : Sirius : 3.3 ^{m} star are in the ratio 1:10^{-10}:10^{-12}.

To determine Δ and d_o it is necessary to measure the stellar eigenmodes, and to resolve them in order to be able to measure them sufficiently accurately. This requires that each of three distinct, and fundamental (and physically-based) observational criteria are met: (a) that each mode is measured with a signal-to-noise ratio, $S/N > 4$: many claims of the detection of stellar modes have been made over the the last 17 years, largely with low S/N data and complex data analysis. Not one detection, to date has, however, been substantiated. (b) that the uncertainty principle is satisfied – if we assume that the oscillations have a lifetime greater than the observation time, then to measure with an error of $1(0.3)\mu Hz$, with a commensurate error in stellar age of 1 (0.3)Ga, requires 2 (6) weeks. (c) that the Nyquist sampling theorem is satisfied – at least 2 measurements need to be made in the shortest time period (highest eigenfrequency) of interest.

Each of the above need to be met for observations that are shorter than the lifetime (coherence time) of the mode. If the observations are longer than the coherence time of the mode, then the power spectra of observations separated by more than the coherence time are independent and gain of accuracy with time is slow. If incoherent, the frequency errors in the power spectrum scale only as the square root of time.

As an additional constraint, it is clear that the instrumental stability and any resultant systematic errors must not be larger than the statistical contribution to the noise levels.

4.1. Photometry from Space

To achieve the required photometric accuracy of 2 parts per million, roughly equivalent to a velocity amplitude of $10cms^{-1}$ (Isaak, 1980), is extremely difficult, or even impossible, from ground level sites. Such accuracy, however, can be readily achieved from a platform above most, or all, of the atmosphere. Whilst an expensive solution, the feasibility of such photometry was amply confirmed when solar oscillations were detected in 1982 at the predicted level (Isaak 1980) with ACRIM (Active Cavity Radiometer) on the Solar Maximum Mission (Woodard & Hudson 1983). A minimum of $4\mathrm{x}10^{12}$ photons has to be detected, and systematic errors have to be correspondingly small. A telescope of $1(0.3)m$ diameter collects such an integrated flux in just over 1 (c. 20) day(s) from an 8th magnitude star.

4.2. Ground-based Spectroscopy

Stars subtend angles of less than some 10 milliarcsec, a factor of 10^{-5} of the Sun. Thus, effects of differential extinction across the stellar disk due to the terrestrial atmosphere, a source of significant systematic error in measurements of global solar oscillations, are negligible. By using differential measurement methods and rapid switching, spectroscopic techniques can be used to reduce systematic errors substantially, again as demonstrated by work on the unresolved Sun over more than two decades. By switching between the blue and the red wings of a spectral line, 'common mode' effects are reduced substantially. Photon noise is, however, a severe problem as the instantaneous fractional bandwidth of spectroscopic instruments is usually small. In principle, we could consider three different scenarios of spectroscopic measurement: (a) a portion of a spectral line

of bandwidth of 50 $m\mathring{A}$ (b) all of one spectral line of $0.2\mathring{A}$ width, for a star that is rotating slowly (c) the measurement of a number, n , of spectral lines. Statistically, we gain a factor of two in going from (a) to (b), and by an additional factor of $\sqrt{n}$ by going to (c). Whether systematic errors can be kept down to correspondingly low values in (c) is unclear. If we consider an ideal spectrometer, with a throughput of unity, with bandwidth as specified by case (a), fed by a 3 (10) metre-diameter flux collector, a total of 7x10^4 (7x10^5) photons per second would be collected from a star of 3.3^m. The latter counting rate is within a factor of two of the Mk I optical resonance scattering spectrometer viewing the Sun (Brookes *et al.*, 1978) of BiSON on Tenerife and was one of the two spectrometers separated spatially by some 2300 km, with which global solar oscillations were first discovered in 1979 (Claverie *et al.*, 1979). Clearly, such a flux collector - ideal spectrometer of type (a), i.e. even of $0.05\mathring{A}$ bandwidth, can readily detect stellar oscillations with typical solar amplitudes.
We propose to discuss another specific spectroscopic system - the magneto-optical-filter (MOF). The MOF was developed by Oehman (1956) and by Cimino, Cacciani & Sopranzi (1968) . An improved version of the MOF (Isaak & Jones, 1986) has been used repeatedly and has reached completely photon noise limited velocity noise at the $35cms^{-1}$ level (Bedford *et al.*, 1995). The throughput of that spectrometer is down on the ideal by about 400. Use of two polarisations (gain: *2), a CCD as a detector rather than an avalanche photodiode (gain: *4), and the use of NaD1 and NaD2, as well as the potassium resonance line at $7699\mathring{A}$ restores a factor of 24. The solar spectral line has a slope which is a factor of *4 steeper. The overall expected performance, scaling from the ACTUAL PERFORMANCE, with a gain of 4x$\sqrt{24}$ improvement on the Bedford *et al.* figure would be a velocity noise level of $2cms^{-1}$ on a 0.3^m star with two weeks observing time on a 1.9 m diameter telescope, or $8cms^{-1}$ for a 3.3^m star. Clearly, flux collectors of the Hanbury-Brown type with modern electronic active optics, and diameters of 6 metres would make the above possible. It should be stressed that the optical resonance spectrometers have an enormous etendue and can, therefore, accept beams from poor quality, i.e. cheaply made, flux collectors.

5. Summary

- Global solar oscillations probe a $1M\odot$ Pop-I star at 4.51 Ga, and so *calibrate* the aeon glass at one point in time, mass and composition. Another point, the zero-age point, may be provided by stars which are theoretically reckoned to be extremely young.

- Stellar oscillations of main-sequence stars could calibrate aeon glasses over a range of ages.

- Both photometry from space and spectroscopy from the ground provide sufficient sensitivity to render the detection of stellar oscillations feasible.

- By measuring the large and the small spacings of the acoustic eigenmode spectrum of old solar-like stars their ages could be measured to substantially better than 1 Ga; these 'internal' measurements could then be used to calibrate the 'external' characteristics of the same stars. Transferring

this calibration, using 'external' characteristics to more distant stars, population I as well as II, then allows the ages of old open and globular clusters to be determined.

- Together with the present value of H_o=67 kms^{-1}Mpc^{-1}, $\Omega = 1$, stellar ages can be used to check on the existence of a cosmological constant Λ. $\Lambda = 0$ demands an age of the Universe of 9.6 Ga, whereas the currently fashionable $\Omega_\Lambda = 0.7$ requires an age of the Universe of 13.8 Ga. Given that any stellar ages provide at best a lower bound to the age of the Universe, if any star were found to have an age in excess of 10 Ga, then stellar seismology would provide an independent confirmation of the need for a non-zero cosmological constant. Stellar ages below 10 Ga would, in contrast, provide necessary but not sufficient evidence to exclude a finite Λ if the oldest stars in our galaxy were to be considerably younger than the Universe.

- We stress that in contrast to the use of SNIa and other techniques, seismological measurements can thus be considered to make possible cosmology studies at zero redshift.

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